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تصحيح التمرين 1:

(1

$$\begin{aligned} I &= \int_0^1 \left(x^2 - x + 1\right)dx \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1 \\ &= \left(\frac{1^3}{3} - \frac{1^2}{2} + 1\right) - \left(\frac{0^3}{3} - \frac{0^2}{2} + 0\right) \\ &= \frac{1}{3} - \frac{1}{2} + 1 \\ &= \frac{5}{6} \end{aligned}$$

(2

$$\begin{aligned} J &= \int_0^{\frac{\pi}{2}} \sin(2x)\,dx \\ &= \left[\frac{-1}{2}\cos(2x)\right]_0^{\frac{\pi}{2}} \\ &= \left(\frac{-1}{2}\cos(\pi)\right) - \left(\frac{-1}{2}\cos(0)\right) \\ &= \left(\frac{-1}{2}(-1)\right) - \left(\frac{-1}{2}(1)\right) \\ &= 1 \end{aligned}$$

(3

$$\begin{aligned} K &= \int_0^{\ln(2)} e^x\,dx \\ &= \left[e^x\right]_0^{\ln(2)} \\ &= \left(e^{\ln(2)}\right) - \left(e^0\right) \\ &= (2) - (1) \\ &= 1 \end{aligned}$$

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(4

$$\begin{aligned}
L &= \int_1^2 \frac{1}{x+1}dx \\
&= \int_1^2 \frac{(x+1)'}{x+1}dx \\
&= \left[\ln|x+1|\right]_1^2 \\
&= (\ln(3))-(\ln(2)) \\
&= \ln\left(\frac{3}{2}\right)
\end{aligned}$$

(5

$$\begin{aligned}
M &= \int_0^4 \sqrt{x+3}dx \\
&= \int_0^4 (x+3)^{\frac{1}{2}}dx \\
&= \int_0^4 (x+3)'(x+3)^{\frac{1}{2}}dx \\
&= \left[\frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 \\
&= \left[\frac{2}{3}(x+3)^{\frac{3}{2}}\right]_0^4 \\
&= \left(\frac{2}{3}(7)^{\frac{3}{2}}\right)-\left(\frac{2}{3}(3)^{\frac{3}{2}}\right) \\
&= \frac{2}{3}\left(7\sqrt{7}-3\sqrt{3}\right)
\end{aligned}$$

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(6

$$\begin{aligned}
N &= \int_0^1 x\left(x^2-1\right)^4 dx \\
&= \frac{1}{2}\int_0^1 (2x)\left(x^2-1\right)^4 dx \\
&= \frac{1}{2}\int_0^1 \left(x^2-1\right)'\left(x^2-1\right)^4 dx \\
&= \frac{1}{2}\left[\frac{\left(x^2-1\right)^5}{5}\right]_0^1 \\
&= \frac{1}{2}\left(\frac{0^5}{5}-\frac{(-1)^5}{5}\right) \\
&= \frac{1}{2}\times\frac{1}{5} \\
&= \frac{1}{10}
\end{aligned}$$

تصحيح التمرين 2:

(1

x	$-\infty$ 1 $+\infty$
$x-1$	$-$ 0 $+$

$$\begin{aligned}
I &= \int_0^2 |x-1| dx \\
&= \int_0^1 |x-1| dx + \int_1^2 |x-1| dx \\
&= \int_0^1 (-x+1) dx + \int_1^2 (x-1) dx \\
&= \left[\frac{-x^2}{2}+x\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^2 \\
&= =\left(\left(\frac{1}{2}\right)-(0)\right)+\left((0)-\left(\frac{-1}{2}\right)\right) \\
&= 1
\end{aligned}$$

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(2

x	0		1		$+\infty$
$ln(x)$		$-$	0	$+$	

$$\begin{aligned}
J &= \int_{\frac{1}{e}}^{e} \frac{|\ln x|}{x} dx \\
&= \int_{\frac{1}{e}}^{1} \frac{|\ln x|}{x} dx + \int_{1}^{e} \frac{|\ln x|}{x} dx \\
&= \int_{\frac{1}{e}}^{1} \frac{-\ln x}{x} dx + \int_{1}^{e} \frac{\ln x}{x} dx \\
&= -\int_{\frac{1}{e}}^{1} \frac{1}{x} \ln x dx + \int_{1}^{e} \frac{1}{x} \ln x dx \\
&= -\int_{\frac{1}{e}}^{1} \ln'(x) \ln x dx + \int_{1}^{e} \ln'(x) \ln x dx \\
&= -\left[\frac{\ln^2(x)}{2}\right]_{\frac{1}{e}}^{1} + \left[\frac{\ln^2(x)}{2}\right]_{1}^{e} + \\
&= -\left((0) - \frac{(-1)^2}{2}\right) + \left(\frac{\ln^2(2)}{2} - (0)\right) \\
&= \frac{1 + \ln^2(2)}{2}
\end{aligned}$$

تصحيح التمرين 3:

(1

$$\begin{aligned}
\mu &= \frac{1}{\frac{\pi}{4} - 0} \int_{0}^{\frac{\pi}{1}} \cos(2x) dx \\
&= \frac{4}{\pi} \left[\frac{1}{2} \sin(2x)\right]_{0}^{\frac{\pi}{4}} \\
&= \frac{4}{\pi} \left(\frac{1}{2} \sin\left(\frac{\pi}{2}\right) - \frac{1}{2} \sin(0)\right) \\
&= \frac{4}{\pi} \left(\frac{1}{2} - 0\right) \\
&= \frac{2}{\pi}
\end{aligned}$$

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2) أ. بما أن $x \mapsto \dfrac{x^4}{1+x^2}$ متصلة و موجبة على $[1,2]$ (و $1<2$) فإن $\displaystyle\int_1^2 \frac{x^4}{1+x^2}dx \geq 0$

ب. بما أن $x \mapsto \ln(x)$ متصلة و سالبة على $\left[\dfrac{1}{e},1\right]$ (و $\dfrac{1}{e}<1$) فإن $\displaystyle\int_{\frac{1}{e}}^1 \ln(x)dx \leq 0$

تصحيح التمرين 4 :

(1

$$\begin{aligned} I &= \int_0^1 x\left(x^2+3\right)^2 dx \\ &= \frac{1}{2}\int_0^1 (2x)\left(x^2+3\right)^2 dx \\ &= \frac{1}{2}\int_0^1 \left(x^2+3\right)'\left(x^2+3\right)^2 dx \\ &= \frac{1}{2}\left[\frac{\left(x^2+3\right)^3}{3}\right]_0^1 \\ &= =\frac{1}{2}\left(\frac{64}{3}-\frac{27}{3}\right) \\ &= \frac{37}{6} \end{aligned}$$

(2

$$\begin{aligned} J &= \int_0^1 \frac{x-1}{\left(x^2-2x+3\right)^2}dx \\ &= \frac{1}{2}\int_0^1 \frac{2x-2}{\left(x^2-2x+3\right)^2}dx \\ &= \frac{1}{2}\int_0^1 \frac{\left(x^2-2x+3\right)'}{\left(x^2-2x+3\right)^2}dx \\ &= \frac{1}{2}\left[\frac{-1}{x^2-2x+3}\right]_0^1 \\ &= =\frac{1}{2}\left(\left(\frac{-1}{2}\right)-\left(\frac{-1}{3}\right)\right) \\ &= \frac{-1}{12} \end{aligned}$$

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(3

$$\begin{aligned} K &= \int_0^2 \frac{2}{x+1}dx \\ &= 2\int_0^2 \frac{1}{x+1}dx \\ &= 2\int_0^2 \frac{(x+1)'}{x+1}dx \\ &= 2\left[\ln|x+1|\right]_0^2 \\ &= =2\left(\left(\ln(3)\right)-\left(\ln(1)\right)\right) \\ &= 2\ln(3) \end{aligned}$$

(4

$$\begin{aligned} L &= \int_1^2 \frac{x}{x+1}dx \\ &= \int_1^2 \frac{x+1-1}{x+1}dx \\ &= \int_1^2 \left(1-\frac{1}{x+1}\right)dx \\ &= \int_1^2 \left(1-\frac{(x+1)'}{x+1}\right)dx \\ &= \left[x-\ln|x+1|\right]_1^2 \\ &= \left(2-\ln(3)\right)-\left(1-\ln(2)\right) \\ &= 1+\ln(2)-\ln(3) \\ &= 1+\ln\left(\frac{2}{3}\right) \end{aligned}$$

(5

$$\begin{aligned} M &= \int_1^e \frac{\ln^2(x)}{x}dx \\ &= \int_1^e \frac{1}{x}\ln^2(x)dx \\ &= \int_1^e \ln'(x)\ln^2(x)dx \\ &= \left[\frac{\ln^3(x)}{3}\right]_1^e \\ &= \frac{\ln^3(e)}{3}-\frac{\ln^3(1)}{3} \\ &= \frac{1}{3} \end{aligned}$$

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تصحيح التمرين 5 :

1) لنحسب $\int_1^e x\ln x dx$.

$$\begin{cases} U'(x)=x \\ V(x)=\ln(x) \end{cases} \nearrow \begin{cases} U(x)=\dfrac{x^2}{2} \\ V'(x)=\dfrac{1}{x} \end{cases} \updownarrow$$

$$\begin{aligned}
\int_1^e x\ln x dx &= \int_1^e U'(x)V(x)dx \\
&= \left[U(x)V(x)\right]_1^e - \int_1^e U(x)V'(x)dx \\
&= \left[\frac{x^2}{2}\ln(x)\right]_1^e - \int_1^e \frac{x^2}{2}\times\frac{1}{x}dx \\
&= \left[\frac{x^2}{2}\ln(x)\right]_1^e - \frac{1}{2}\int_1^e x dx \\
&= \left[\frac{x^2}{2}\ln(x)\right]_1^e - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^e \\
&= \left(\frac{e^2}{2}\ln(e) - \frac{1^2}{2}\ln(1)\right) - \frac{1}{2}\left(\frac{e^2}{2} - \frac{1^2}{2}\right) \\
&= \frac{e^2}{2} - \frac{e^2}{4} + \frac{1}{4} \\
&= \frac{e^2+1}{4}
\end{aligned}$$

2) لنحسب $\int_0^{\frac{\pi}{2}} x\cos x dx$.

$$\begin{cases} U'(x)=\cos x \\ V(x)=x \end{cases} \nearrow \begin{cases} U(x)=\sin x \\ V'(x)=1 \end{cases} \updownarrow$$

$$\begin{aligned}
\int_0^{\frac{\pi}{2}} x\cos x dx &= \left[x\sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx \\
&= \left[x\sin x\right]_0^{\frac{\pi}{2}} - \left[-\cos x\right]_0^{\frac{\pi}{2}} \\
&= \left(\left(\frac{\pi}{2}\right) - (0)\right) - \left((0) - (-1)\right) \\
&= \frac{\pi}{2} - 1
\end{aligned}$$

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3) لنحسب $\int_1^e \ln x dx$.

لدينا : $\int_1^e \ln x dx = \int_1^e 1\times\ln(x)dx$

$$\begin{cases} U'(x)=1 \\ V(x)=\ln x \end{cases} \quad \swarrow\!\!\nearrow \quad \begin{cases} U(x)=x \\ V'(x)=\dfrac{1}{x} \end{cases} \updownarrow$$

$$\begin{aligned}\int_1^e \ln x dx &= [x\ln x]_1^e - \int_1^e x\times\frac{1}{x}dx \\ &= [x\ln x]_1^e - \int_1^e 1dx \\ &= [x\ln x]_1^e - [x]_1^e \\ &= (e-0)-(e-1) \\ &= 1\end{aligned}$$

4) لنحسب $\int_0^1 \ln(x+1)dx$.

لدينا : $\int_0^1 \ln(x+1)dx = \int_0^1 1\times\ln(x+1)dx$

$$\begin{cases} U'(x)=1 \\ V(x)=\ln(x+1) \end{cases} \quad \swarrow\!\!\nearrow \quad \begin{cases} U(x)=x+1 \\ V'(x)=\dfrac{(x+1)'}{x+1}=\dfrac{1}{x+1} \end{cases} \updownarrow$$

$$\begin{aligned}\int_0^1 \ln(x+1)dx &= \left[(x+1)\ln(x+1)\right]_0^1 - \int_0^1 (x+1)\times\frac{1}{x+1}dx \\ &= \left[(x+1)\ln(x+1)\right]_0^1 - \int_0^1 1dx \\ &= \left[(x+1)\ln(x+1)\right]_0^1 - [x]_0^1 \\ &= (2\ln(2)-0)-(1-0) \\ &= 2\ln(2)-1\end{aligned}$$

5) لنحسب $\int_e^{e^2} \frac{\ln x}{\sqrt{x}}dx$:

لدينا : $\int_e^{e^2} \frac{\ln x}{\sqrt{x}}dx = \int_e^{e^2} \frac{1}{\sqrt{x}}\times\ln x dx$

$$\begin{cases} U'(x)=\dfrac{1}{\sqrt{x}} \\ V(x)=\ln(x) \end{cases} \quad \swarrow\!\!\nearrow \quad \begin{cases} U(x)=2\sqrt{x} \\ V'(x)=\dfrac{1}{x} \end{cases} \updownarrow$$

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$$\int_e^{e^2}\frac{\ln x}{\sqrt{x}}dx = \left[2\sqrt{x}\ln x\right]_e^{e^2} - \int_e^{e^2} 2\sqrt{x}\times\frac{1}{x}dx$$
$$= \left[2\sqrt{x}\ln x\right]_e^{e^2} - 2\int_e^{e^2}\frac{1}{\sqrt{x}}dx$$
$$= \left[2\sqrt{x}\ln x\right]_e^{e^2} - 2\left[2\sqrt{x}\right]_e^{e^2}$$
$$= \left(2\sqrt{e^2}\ln\left(e^2\right) - 2\sqrt{e}\ln e\right) - 2\left(2\sqrt{e^2} - 2\sqrt{e}\right)$$
$$= \left(4e - 2\sqrt{e}\right) - 2\left(2e - 2\sqrt{e}\right)$$
$$= 4e - 2\sqrt{e} - 4e + 4\sqrt{e}$$
$$= 2\sqrt{e}$$

6) لنحسب $\int_0^1 x^2 e^x dx$:

$$\begin{cases} U'(x) = e^x \\ V(x) = x^2 \end{cases} \nearrow \quad \begin{cases} U(x) = e^x \\ V'(x) = 2x \end{cases} \updownarrow$$

$$\int_0^1 x^2 e^x dx = \left[x^2 e^x\right]_0^1 - \int_0^1 2x e^x dx$$
$$= \left[x^2 e^x\right]_0^1 - 2\int_0^1 x e^x dx$$
$$= e - 2\int_0^1 x e^x dx$$

لنحسب $\int_0^1 x e^x dx$:

$$\begin{cases} h'(x) = e^x \\ g(x) = x \end{cases} \nearrow \quad \begin{cases} h(x) = e^x \\ g'(x) = 1 \end{cases} \updownarrow$$

$$\int_0^1 x e^x dx = \left[x e^x\right]_0^1 - \int_0^1 e^x dx$$
$$= \left[x e^x\right]_0^1 - \left[e^x\right]_0^1$$
$$= (e - 0) - (e - 1) = 1$$

و بالتالي : $\int_0^1 x^2 e^x dx = e - 2$

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تصحيح التمرين 6

(1

$$\begin{aligned} A &= \int_1^4 |f(x)|dx.(U.A) \\ &= \int_1^4 \left|\frac{2}{x}\right|dx.(U.A) \\ &= \int_1^4 \frac{2}{x}dx.(U.A) \\ &= 2\int_1^4 \frac{1}{x}dx.(U.A) \\ &= 2[\ln x]_1^4.(U.A) \\ &= 2(\ln(4)-\ln(1)).(U.A) \\ &= 2\ln(4).(U.A) \end{aligned}$$

(2

$$\begin{aligned} A &= \int_{\ln 2}^{\ln 4} |f(x)|dx.\|\vec{i}\|.\|\vec{j}\| \\ &= \int_{\ln 2}^{\ln 4} |1-e^x|dx.2cm.2cm \\ &= \int_{\ln 2}^{\ln 4} (e^x-1)dx.4cm^2 \\ &= [e^x-x]_{\ln 2}^{\ln 4}.4cm^2 \\ &= ((4-\ln 4)-(2-\ln 2)).4cm^2 \\ &= (2-\ln 2).4cm^2 \\ &= (8-4\ln(2))cm^2 \end{aligned}$$

(3

$$\begin{aligned} A &= \int_1^3 |f(x)|dx.\|\vec{i}\|.\|\vec{j}\| \\ &= \int_1^3 |x^2-2x|dx.1cm.1cm \\ &= \left(\int_1^2 |x^2-2x|dx + \int_2^3 |x^2-2x|dx\right)cm^2 \\ &= \left(\int_1^2 (2x-x^2)dx + \int_2^3 (x^2-2x)dx\right)cm^2 \\ &= \left(\left[x^2-\frac{x^3}{3}\right]_1^2 + \left[\frac{x^3}{3}-x^2\right]_2^3\right)cm^2 \\ &= \left(\left(4-\frac{8}{3}\right)-\left(1-\frac{1}{3}\right)+(9-9)-\left(\frac{8}{3}-4\right)\right)cm^2 \\ &= 2cm^2 \end{aligned}$$

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(4

$$\begin{aligned} A &= \int_0^{\ln 2} \left| f(x) - g(x) \right| dx . \|\vec{i}\| . \|\vec{j}\| \\ &= \int_0^{\ln 2} \left| \frac{2e^x}{e^x+1} \right| dx .2cm.3cm \\ &= \int_0^{\ln 2} \left(\frac{2e^x}{e^x+1} \right) dx .6cm^2 \\ &= 2\int_0^{\ln 2} \left(\frac{(e^x+1)'}{e^x+1} \right) dx .6cm^2 \\ &= 12.\left[\ln\left|e^x+1\right| \right]_0^{\ln 2} .cm^2 \\ &= 12\left(\ln(3) - \ln(2)\right).cm^2 \\ &= 12\ln\left(\frac{3}{2}\right) cm^2 \end{aligned}$$

تصحيح التمرين 7:

$$\begin{aligned} V &= \int_0^1 \pi f^2(x) dx .(U.V) \\ &= \pi \int_0^1 x\left(e^x - 1\right) dx . \\ &= \pi \int_0^1 \left(xe^x - x\right) dx .(U.V) \\ &= \pi \left(\int_0^1 xe^x dx - \int_0^1 x dx \right).(U.V) \\ &= \pi \left(1 - \left[\frac{x^2}{2} \right]_0^1 \right).(U.V) \\ &= \pi \left(1 - \left(\frac{1}{2} \right) \right).(U.V) \\ &= \frac{\pi}{2}(U.V) \end{aligned}$$

(بالنسبة للتكامل $\int_0^1 xe^x dx$ انظر تصحيح التمرين 5 السؤال 6 باستعمال مكاملة بالأجزاء : $\int_0^1 xe^x dx = 1$)

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تصحيح التمرين 8:

(1

$$\int_1^2\left(x^4-\frac{1}{4}x^3+2x-5-\frac{1}{x}+\frac{4}{\sqrt{x}}\right)dx \quad =\left[\frac{x^5}{5}-\frac{x^4}{16}+x^2-5x+8\sqrt{x}\right]_1^2$$

$$=8\sqrt{2}-\ln(2)-\frac{379}{80}$$

(2

$$\int_0^1 3x\left(x^2-1\right)^4dx \quad =\frac{3}{2}\int_0^1 2x\left(x^2-1\right)^4dx$$

$$=\frac{3}{2}\int_0^1\left(x^2-1\right)'\left(x^2-1\right)^4dx$$

$$=\frac{3}{2}\left[\frac{\left(x^2-1\right)^5}{5}\right]_0^1$$

$$=\frac{3}{10}$$

(3

$$\int_0^1\frac{x^2}{\sqrt{x^3+1}}dx \quad =\frac{1}{3}\int_0^1\frac{3x^2}{\sqrt{x^3+1}}dx$$

$$=\frac{1}{3}\int_0^1\frac{\left(x^3+1\right)'}{\sqrt{x^3+1}}dx$$

$$=\frac{1}{3}\left[2\sqrt{x^3+1}\right]_0^1$$

$$=\frac{1}{3}\left(2\sqrt{2}-2\right)$$

$$=\frac{2}{3}\left(\sqrt{2}-1\right)$$

(4

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$$\int_0^1 \frac{x^5}{\sqrt[3]{x^6+1}}dx = \frac{1}{6}\int_0^1 6x^5\left(x^6+1\right)^{\frac{-1}{3}}dx$$

$$= \frac{1}{6}\int_0^1 \left(x^6+1\right)'\left(x^6+1\right)^{\frac{-1}{3}}dx$$

$$= \frac{1}{6}\left[\frac{\left(x^6+1\right)^{\frac{2}{3}}}{\frac{2}{3}}\right]_0^1$$

$$= \frac{1}{6}\left[\frac{3}{2}\sqrt[3]{\left(x^6+1\right)^2}\right]_0^1$$

$$= \frac{1}{6}\left(\frac{3}{2}\sqrt[3]{4}-\frac{3}{2}\right)$$

$$= \frac{\sqrt[3]{4}-1}{4}$$

(5
$$\int_1^e \frac{\ln^2(x)}{x}dx = \int_1^e \ln'(x)\ln^2(x)dx = \left[\frac{\ln^3(x)}{3}\right]_1^e = \frac{1}{3}$$

(6
$$\int_2^3 \frac{x^3}{\left(x^4-1\right)^2}dx = \frac{1}{4}\int_2^3 \frac{4x^3}{\left(x^4-1\right)^2}dx$$

$$= \frac{1}{4}\int_2^3 \frac{\left(x^4-1\right)'}{\left(x^4-1\right)^2}dx$$

$$= \frac{1}{4}\left[\frac{-1}{x^4-1}\right]_2^3$$

$$= \frac{19}{960}$$

(7
$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int_1^4 \left(\sqrt{x}\right)'e^{\sqrt{x}}dx = 2\left[e^{\sqrt{x}}\right]_1^4 = 2\left(e^2-e\right)$$

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(8

$$\int_0^1 \frac{x+2}{x^2+4x+3}dx = \frac{1}{2}\int_0^1 \frac{2x+4}{x^2+4x+3}dx$$
$$= \frac{1}{2}\int_0^1 \frac{\left(x^2+4x+3\right)'}{x^2+4x+3}dx$$
$$= \frac{1}{2}\left[\ln\left|x^2+4x+3\right|\right]_0^1$$
$$= \frac{1}{2}\left(\ln 8 - \ln 3\right)$$
$$= \frac{1}{2}\ln\left(\frac{8}{3}\right)$$

تصحيح التمرين 9:

1) لنحسب : $\int_0^1 (x+2)e^{-x}dx$

$$\begin{cases} U'(x) = e^{-x} \\ V(x) = x+2 \end{cases} \nearrow \quad \begin{cases} U(x) = -e^{-x} \\ V'(x) = 1 \end{cases} \updownarrow$$

$$\int_0^1 (x+2)e^{-x}dx = \left[-(x+2)e^{-x}\right]_0^1 - \int_0^1 -e^{-x}dx$$
$$= \left[-(x+2)e^{-x}\right]_0^1 - \left[-e^{-x}\right]_0^1$$
$$= \left(-3e^{-1}\right) - (-2) - \left(e^{-1} - 1\right)$$
$$= 3 - 4e^{-1}$$

2) لنحسب : $\int_0^1 x\ln(x+3)dx$

$$\begin{cases} U'(x) = x \\ V(x) = \ln(x+3) \end{cases} \nearrow \quad \begin{cases} U(x) = \frac{x^2}{2} \\ V'(x) = \frac{1}{x+3} \end{cases} \updownarrow$$

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$$\begin{aligned}\int_0^1 x\ln(x+3)dx &= \left[\frac{x^2}{2}\ln(x+3)\right]_0^1 - \frac{1}{2}\int_0^1 \frac{x^2}{x+3}dx \\ &= \left(\frac{1}{2}\ln 4\right) - (0) - \frac{1}{2}\int_0^1 \frac{x^2-3^2+3^2}{x+3}dx \\ &= \frac{1}{2}\ln 4 - \frac{1}{2}\int_0^1 \frac{(x-3)(x+3)+9}{x+3}dx \\ &= \frac{1}{2}\ln 4 - \frac{1}{2}\int_0^1 \left(x-3+\frac{9(x+3)'}{x+3}\right)dx \\ &= \frac{1}{2}\ln 4 - \frac{1}{2}\left[\frac{x^2}{2} - 3x + 9\ln|x+3|\right]_0^1 \\ &= \frac{1}{2}\ln 4 - \left(\left(\frac{-5}{2} + 9\ln 4\right) - (9\ln 3)\right) \\ &= \frac{5}{4} - 4\ln 4 + \frac{9}{2}\ln 3\end{aligned}$$

(3

أ) ليكن x من $\mathbb{R}\setminus\{-1;0;1\}$

$$\begin{aligned}\frac{-1}{x} + \frac{x}{x^2-1} &= \frac{-(x^2-1)+x^2}{x(x^2-1)} \\ &= \frac{-x^2+1+x^2}{x(x^2-1)}\end{aligned}$$

إذن : لكل x من $\mathbb{R}\setminus\{-1;0;1\}$ لدينا : $\frac{1}{x(x^2-1)} = \frac{-1}{x} + \frac{x}{x^2-1}$

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ب)

$$\int_2^3 \frac{1}{x\left(x^2-1\right)}dx \qquad = \int_2^3 \left(\frac{-1}{x}+\frac{x}{x^2-1}\right)dx$$

$$= \int_2^3 \left(\frac{-1}{x}+\frac{1}{2}\times\frac{2x}{x^2-1}\right)dx$$

$$= \int_2^3 \left(\frac{-1}{x}+\frac{1}{2}\times\frac{\left(x^2-1\right)'}{x^2-1}\right)dx$$

$$= \left[-\ln x+\frac{1}{2}\ln\left|x^2-1\right|\right]_2^3$$

$$= \left(-\ln 3+\frac{1}{2}\ln 8\right)-\left(-\ln 2+\frac{1}{2}\ln 3\right)$$

$$= \frac{-3}{2}\ln 3+\frac{5}{2}\ln 2$$

ج) لنحسب : $\int_2^3 \frac{x}{\left(x^2-1\right)^2}\ln(x)dx$

$$\begin{cases} U'(x)=\dfrac{x}{\left(x^2-1\right)^2}=\dfrac{1}{2}\times\dfrac{2x}{\left(x^2-1\right)^2}=\dfrac{\left(x^2-1\right)'}{\left(x^2-1\right)^2} \\ V(x)=\ln(x) \end{cases} \nearrow\!\!\!\swarrow \begin{cases} U(x)=\dfrac{-1}{2\left(x^2-1\right)} \\ V'(x)=\dfrac{1}{x} \end{cases} \updownarrow$$

$$\int_2^3 \frac{x}{\left(x^2-1\right)^2}\ln(x)dx \qquad = \left[\frac{-1}{2\left(x^2-1\right)}\ln x\right]_2^3 - \int_2^3 \frac{-1}{2x\left(x^2-1\right)}dx$$

$$= \left(\frac{-1}{16}\ln 3\right)-\left(\frac{-1}{6}\ln 2\right)+\frac{1}{2}\int_2^3 \frac{1}{x\left(x^2-1\right)}dx$$

$$= \frac{-1}{16}\ln 3+\frac{1}{6}\ln 2-\frac{3}{4}\ln 3+\frac{5}{4}\ln 2$$

$$= \frac{17}{12}\ln 2-\frac{13}{16}\ln 3$$

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تصحيح التمرين 10:

1) لنحسب $\int_0^1 xe^x dx$:

$$\begin{cases} h'(x)=e^x \\ g(x)=x \end{cases} \swarrow\!\!\!\nearrow \begin{cases} h(x)=e^x \\ g'(x)=1 \end{cases} \updownarrow$$

$$\begin{aligned}\int_0^1 xe^x dx &= \left[xe^x\right]_0^1 - \int_0^1 e^x dx \\ &= \left[xe^x\right]_0^1 - \left[e^x\right]_0^1 \\ &= (e-0)-(e-1)=1\end{aligned}$$

2) لنحسب $\int_0^1 x^2e^x dx$:

$$\begin{cases} U'(x)=e^x \\ V(x)=x^2 \end{cases} \swarrow\!\!\!\nearrow \begin{cases} U(x)=e^x \\ V'(x)=2x \end{cases} \updownarrow$$

$$\begin{aligned}\int_0^1 x^2e^x dx &= \left[x^2e^x\right]_0^1 - \int_0^1 2xe^x dx \\ &= \left[x^2e^x\right]_0^1 - 2\int_0^1 xe^x dx \\ &= e-2\int_0^1 xe^x dx\end{aligned}$$

و منه : $\int_0^1 x^2e^x dx = e-2$

3)

$$\begin{aligned}A &= \int_0^1 |f(x)|dx \times \|\vec{i}\| \times \|\vec{j}\| \\ &= \int_0^1 (x^2+4x+4)e^x dx \times 1{,}5cm \times 2cm \qquad \left((x^2+4x+4)e^x = (x+2)^2 e^x \geq 0\right) \\ &= \left(\int_0^1 x^2e^x dx + 4\int_0^1 xe^x dx + 4\int_0^1 e^x dx\right) \times 3cm^2 \\ &= \left(e-2+4\times1+4\left[e^x\right]_0^1\right)\times 3cm^2 \\ &= \left(e+2+4(e-1)\right)\times 3cm^2 \\ &= (5e-2)\times 3cm^2 \\ &= (15e-6)cm^2\end{aligned}$$

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تصحيح التمرين 11:

(1

✓ الدالة F قابلة للإشتقاق على $]0,+\infty[$

✓ ليكن $x \in]0,+\infty[$:

$$\begin{aligned} F'(x) &= \left(\frac{-\ln x}{x} - \frac{1}{x}\right)' \\ &= -\left(\frac{\ln x}{x} + \frac{1}{x}\right)' \\ &= -\left(\frac{\ln'(x)\times x - \ln(x)\times(x)'}{x^2} - \frac{1}{x^2}\right) \\ &= -\frac{\frac{1}{x}\times x - \ln x}{x^2} + \frac{1}{x^2} \\ &= -\frac{1-\ln x}{x^2} + \frac{1}{x^2} \\ &= -\frac{1-\ln x - 1}{x^2} = \frac{\ln x}{x^2} \end{aligned}$$

إذن لكل x من $]0,+\infty[$:

و بالتالي الدالة F دالة أصلية للدالة f على المجال $]0,+\infty[$

(2

$$\begin{aligned} \int_1^4 \frac{\ln x}{x^2}dx &= \left[\frac{-\ln x}{x} - \frac{1}{x}\right]_1^4 \\ &= \left(\frac{-\ln 4}{4} - \frac{1}{4}\right) - (0-1) \\ & \frac{3-\ln 4}{4} \end{aligned}$$

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تصحيح التمرين 12:

1) لنبين أن الدالة $H : x \mapsto x\ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0,+\infty[$

✓ الدالة H قابلة للإشتقاق على $]0,+\infty[$

✓ ليكن $x \in]0,+\infty[$:

$$\begin{aligned} H'(x) &= (x\ln x - x)' \\ &= (x)'\ln x + x\ln'(x) - 1 \\ &= \ln x + x \times \frac{1}{x} - 1 \\ &= \ln(x) + 1 - 1 \\ &= \ln x \end{aligned}$$

إذن لكل x من $]0,+\infty[$: $H'(x) = h(x)$

و منه الدالة $H : x \mapsto x\ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0,+\infty[$

2)

$$\begin{aligned} \int_1^2 h(x)dx &= [H(x)]_1^2 \\ &= H(2) - H(1) \\ &= (2\ln 2 - 2) - (1\ln 1 - 1) \\ &= 2\ln 2 - 2 - 0 + 1 \\ & 2\ln 2 - 1 \end{aligned}$$

3) $A = \int_1^2 |h(x)|dx \times \|\vec{i}\| \times \|\vec{j}\|$

على المجال $[1,2]$ لدينا : $h(x) = \ln x \geq 0$

إذن : $A = \int_1^2 h(x)dx \times 2cm \times 2cm$

إذن : $A = (2\ln 2 - 1) \times 4cm^2$

و منه : $A = (8\ln 2 - 4)cm^2$

تصحيح التمرين 13:

لنحسب : $\int_2^4 |\ln(x) - 1|dx$:

x	0	e	$+\infty$
$ln(x)-1$	$-$	0	$+$

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$$\begin{aligned}\int_2^4 |\ln x - 1|dx &= \int_2^e |\ln x - 1|dx + \int_e^4 |\ln x - 1|dx \\ &= \int_2^e (1-\ln x)dx + \int_e^4 (\ln x - 1)dx \\ &= \left[x - (x\ln x - x)\right]_2^e + \left[(x\ln x - x) - x\right]_e^4 \\ &= \left[2x - x\ln x\right]_2^e + \left[x\ln x - 2x\right]_e^4 \\ &= 2e - 12 + 10\ln 2\end{aligned}$$

(انظر التمرين 12 لدينا الدالة $H : x \mapsto x\ln x - x$ دالة أصلية للدالة $h : x \mapsto \ln x$ على $]0,+\infty[$)

تصحيح التمرين 14:

(1

- لنحسب : $A = \int_1^2 (x^3 - 2x + 3)dx$

$$\begin{aligned}A &= \int_1^2 (x^3 - 2x + 3)dx \\ &= \left[\frac{x^4}{4} - x^2 + 3x\right]_1^2 \\ &= (4 - 4 + 6) - \left(\frac{1}{4} - 1 + 3\right) \\ &= \frac{15}{4}\end{aligned}$$

- لنحسب : $B = \int_0^1 x(x^2+1)^2 dx$

$$\begin{aligned}B &= \int_0^1 x(x^2+1)^2 dx \\ &= \frac{1}{2}\int_0^1 2x(x^2+1)dx \\ &= \frac{1}{2}\int_0^1 (x^2+1)'(x^2+1)dx \\ &= \frac{1}{2}\left[\frac{(x^2+1)^3}{3}\right]_0^1 \\ &= \frac{1}{2}\left(\frac{8}{3} - \frac{1}{3}\right) \\ &= \frac{7}{6}\end{aligned}$$

- لنحسب : $C = \int_0^2 \frac{1}{x+1}dx$

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$$\begin{aligned} C &= \int_0^2 \frac{1}{x+1}dx \\ &= \int_0^2 \frac{(x+1)'}{x+1}dx \\ &= \left[\ln|x+1|\right]_0^2 \\ &= \ln 3 - \ln 1 \\ &= \ln 3 \end{aligned}$$

2) لنحسب : $D = \int_{\frac{1}{e}}^{e} \frac{|\ln x|}{x}dx$

x	0		1	$+\infty$
$ln(x)$		$-$	0	$+$

$$\begin{aligned} \int_{\frac{1}{e}}^{e} \frac{|\ln x|}{x}dx &= \int_{\frac{1}{e}}^{1} \frac{|\ln x|}{x}dx + \int_{1}^{e} \frac{|\ln x|}{x}dx \\ &= \int_{\frac{1}{e}}^{1} -\frac{1}{x}\ln x\,dx + \int_{1}^{e} \frac{1}{x}\ln x\,dx \\ &= -\int_{\frac{1}{e}}^{1} \ln' x \ln x\,dx + \int_{1}^{e} \ln' x \ln x\,dx \\ &= -\left[\frac{\ln^2 x}{2}\right]_{\frac{1}{e}}^{1} + \left[\frac{\ln^2 x}{2}\right]_{1}^{e} \\ &= -\left(0-\frac{1}{2}\right)+\left(\frac{1}{2}-0\right) \\ &= 1 \end{aligned}$$

3)

أ) ليكن $x \in \mathbb{R} \setminus \{-1\}$:

$$\begin{aligned} x-1+\frac{1}{x+1} &= \frac{(x-1)(x+1)+1}{x+1} \\ &= \frac{x^2-1+1}{x+1} \\ &= \frac{x^2}{x+1} \end{aligned}$$

إذن : $(\forall x \in \mathbb{R} \setminus \{-1\}) : \frac{x^2}{x+1} = x-1+\frac{1}{x+1}$

ب) لنحسب $E = \int_0^2 \frac{x^2}{x+1}dx$

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$$\begin{aligned} E &= \int_0^2 \frac{x^2}{x+1}dx \\ &= \int_0^2 \left(x-1+\frac{1}{x+1} \right) dx \\ &= \int_0^2 \left(x-1+\frac{(x+1)'}{x+1} \right) dx \\ &= \left[\frac{x^2}{2} - x + \ln|x+1| \right]_0^2 \\ &= \ln 3 \end{aligned}$$

ج) لنحسب : $F = \int_0^2 x\ln(x+1)dx$

$$\begin{cases} U'(x) = x \\ V(x) = \ln(x+1) \end{cases} \nearrow \swarrow \begin{cases} U(x) = \dfrac{x^2}{2} \\ V'(x) = \dfrac{1}{x+1} \end{cases} \updownarrow$$

$$\begin{aligned} F &= \int_0^2 x\ln(x+1)dx \\ &= \left[\frac{x^2}{2}\ln(x+1) \right]_0^2 - \int_0^2 \frac{x^2}{2(x+1)}dx \\ &= (2\ln(3)-0) - \frac{1}{2}\int_0^2 \frac{x^2}{x+1}dx \\ & 2\ln 3 - \frac{1}{2}E \\ &= 2\ln 3 - \frac{1}{2}\ln 3 = \frac{3}{2}\ln 3 \end{aligned}$$